



Cambridge International AS & A Level

CANDIDATE
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CENTRE
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FURTHER MATHEMATICS

9231/23

Paper 2 Further Pure Mathematics 2

May/June 2022

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Any blank pages are indicated.

- 1 Find the roots of the equation $z^3 = 7\sqrt{3} - 7i$, giving your answers in the form $re^{i\theta}$, where $r > 0$ and $-\pi \leq \theta < \pi$. [5]

- 2 (a) Find the coefficient of x^2 in the Maclaurin's series for $-\ln \cos x$. [4]

- (b) Find the length of the arc of the curve with equation $y = -\ln \cos x$ from the point where $x = 0$ to the point where $x = \frac{1}{4}\pi$. [4]

3 The matrix \mathbf{A} is given by

$$\mathbf{A} = \begin{pmatrix} 6 & -9 & 5 \\ 5 & -8 & 5 \\ 1 & -1 & 2 \end{pmatrix}.$$

- (a) Find the eigenvalues of \mathbf{A} . [4]

- (b) Use the characteristic equation of \mathbf{A} to show that $\mathbf{A}^{-1} = p\mathbf{A}^2 + q\mathbf{I}$, where p and q are constants to be determined. [3]

- 4** It is given that

$$x = -t + \tan^{-1} t \quad \text{and} \quad y = t + \sinh^{-1} t.$$

- (a) Show that $\frac{dy}{dx} = -\frac{t^2 + 1 + \sqrt{t^2 + 1}}{t^2}$. [4]

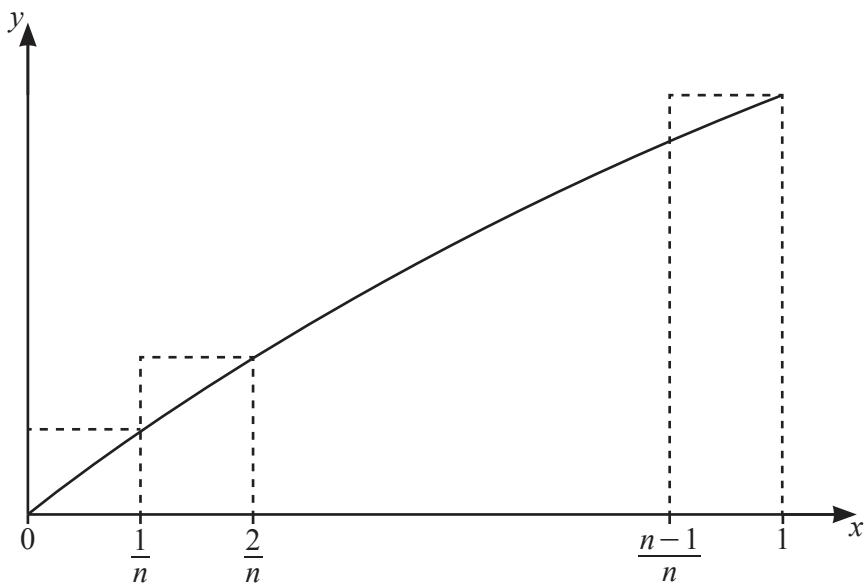
- (b)** Find the value of $\frac{d^2y}{dx^2}$ when $t = \frac{3}{4}$. [5]

- 5** Find the solution of the differential equation

$$x(x+7)\frac{dy}{dx} + 7y = x$$

for which $y = 7$ when $x = 1$. Give your answer in the form $y = f(x)$.

[9]



The diagram shows the curve with equation $y = \ln(1+x)$ for $0 \leq x \leq 1$, together with a set of n rectangles each of width $\frac{1}{n}$.

- (a) By considering the sum of the areas of these rectangles, show that $\int_0^1 \ln(1+x) dx < U_n$, where

$$U_n = \frac{1}{n} \ln \frac{(2n)!}{n!} - \ln n. \quad [4]$$

- (b) Use a similar method to find, in terms of n , a lower bound L_n for $\int_0^1 \ln(1+x)dx$. [4]

- (c) By simplifying $U_n - L_n$, show that $\lim_{n \rightarrow \infty} (U_n - L_n) = 0$. [2]

- 7 The variables x and y are related by the differential equation

$$4 \frac{d^2y}{dx^2} - y = 3.$$

It is given that, when $x = 0$, $y = -3$ and $\frac{dy}{dx} = 2$.

- (a) Find y in terms of x .

[8]

- (b) Deduce the exact value of x for which $y = 0$. Give your answer in logarithmic form. [3]

- 8 (a) Find $\int \sin \theta \cos^n \theta d\theta$, where $n \neq -1$. [2]

$$\text{Let } I_{m,n} = \int_0^{\frac{1}{2}\pi} \sin^m \theta \cos^n \theta d\theta.$$

- (b) Show that, for $m \geq 2$ and $n \geq 0$,

$$I_{m,n} = \frac{m-1}{m+n} I_{m-2,n}. \quad [5]$$

- (c) By considering the binomial expansion of $\left(z + \frac{1}{z}\right)^5$, where $z = \cos \theta + i \sin \theta$, use de Moivre's theorem to show that

$$\cos^5 \theta = a \cos 5\theta + b \cos 3\theta + c \cos \theta,$$

where a , b and c are constants to be determined.

[5]

- (d) Using the results given in parts (b) and (c), find the exact value of $I_{2,5}$. [4]

Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.

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