Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME			
CENTRE NUMBER		CANDIDATE NUMBER	

2464209347

FURTHER MATHEMATICS **9231/23**

Paper 2 Further Pure Mathematics 2 **May/June 2022**

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

DC (PQ/SG) 214267/1

1 Find the roots of the equation $z^3 = 7\sqrt{3} - 7\mathrm{i}$, giving your answers in the form $r\mathrm{e}^{\mathrm{i}\theta}$, where $r > 0$ and $-\pi \leqslant \theta < \pi$. [5]

2 **(a)** Find the coefficient of x^2 in the Maclaurin's series for $-\ln\cos x$. [4]

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(b) Find the length of the arc of the curve with equation $y = -\ln\cos x$ from the point where $x = 0$ to the point where $x = \frac{1}{4}\pi$. [4]

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3 The matrix **A** is given by

$$\mathbf{A} = \begin{pmatrix} 6 & -9 & 5 \\ 5 & -8 & 5 \\ 1 & -1 & 2 \end{pmatrix}.$$

(a) Find the eigenvalues of **A**. [4]

(b) Use the characteristic equation of $\mathbf{A}$ to show that $\mathbf{A}^{-1} = p\mathbf{A}^2 + q\mathbf{I}$, where p and q are constants to be determined. [3]

4 It is given that

$$x = -t + \tan^{-1} t \qquad \text{and} \qquad y = t + \sinh^{-1} t.$$

(a) Show that $\dfrac{dy}{dx} = -\dfrac{t^2 + 1 + \sqrt{t^2+1}}{t^2}$. [4]

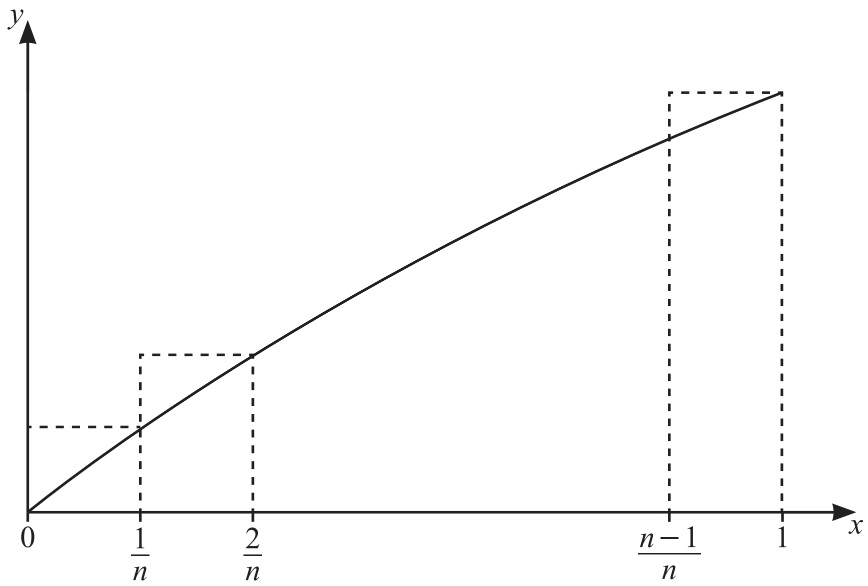
(b) Find the value of $\frac{d^2y}{dx^2}$ when $t = \frac{3}{4}$. [5]

5 Find the solution of the differential equation

$$x(x+7)\frac{dy}{dx}+7y=x$$

for which $y = 7$ when $x = 1$. Give your answer in the form $y = f(x)$. [9]

6



The diagram shows the curve with equation $y = \ln(1+x)$ for $0 \leqslant x \leqslant 1$, together with a set of n rectangles each of width $\frac{1}{n}$.

(a) By considering the sum of the areas of these rectangles, show that $\int_0^1 \ln(1+x)\,dx < U_n$, where

$$U_n = \frac{1}{n}\ln\frac{(2n)!}{n!} - \ln n.$$

[4]

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(b) Use a similar method to find, in terms of n, a lower bound L_n for $\int_0^1 \ln(1+x)\,dx$. [4]

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(c) By simplifying $U_n - L_n$, show that $\lim_{n\to\infty}(U_n - L_n) = 0$. [2]

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7 The variables x and y are related by the differential equation

$$4\frac{d^2y}{dx^2} - y = 3.$$

It is given that, when $x = 0$, $y = -3$ and $\frac{dy}{dx} = 2$.

(a) Find y in terms of x. [8]

(b) Deduce the exact value of x for which $y = 0$. Give your answer in logarithmic form. [3]

8 **(a)** Find $\int \sin\theta \cos^n\theta \, d\theta$, where $n \neq -1$. [2]

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Let $I_{m,n} = \int_0^{\frac{1}{2}\pi} \sin^m\theta \cos^n\theta \, d\theta$.

(b) Show that, for $m \geqslant 2$ and $n \geqslant 0$,

$$I_{m,n} = \frac{m-1}{m+n} I_{m-2,n}.$$ [5]

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(c) By considering the binomial expansion of $\left(z+\frac{1}{z}\right)^5$, where $z = \cos\theta + \mathrm{i}\sin\theta$, use de Moivre's theorem to show that

$$\cos^5\theta = a\cos 5\theta + b\cos 3\theta + c\cos\theta,$$

where a, b and c are constants to be determined. [5]

(d) Using the results given in parts **(b)** and **(c)**, find the exact value of $I_{2,5}$. [4]

Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.

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